Lesson 9: Proportional Control Action

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ET 438A AUTOMATIC CONTROL SYSTEMS TECHNOLOGY

Learning Objectives

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After this presentation you will be able to:

- Identify the components of a proportional feedback control system.
- Write a mathematical model for a proportional controller.
- Compute the proportional bandwidth of a proportional controller.
- Explain the relationship between steady-state error and proportional gain.
- Perform lab experiment 2 more effectively.

Proportional Control Action

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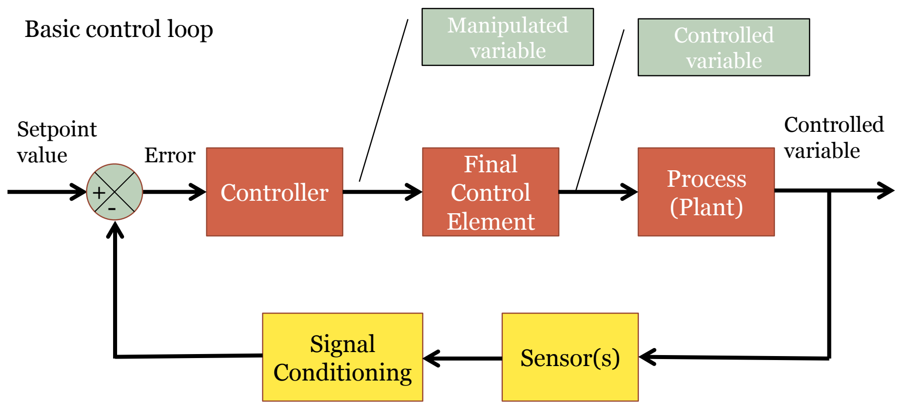
Simplest form of controller - amplify error and apply signal to the process through final control element.

Basic control loop



Proportional Control Action

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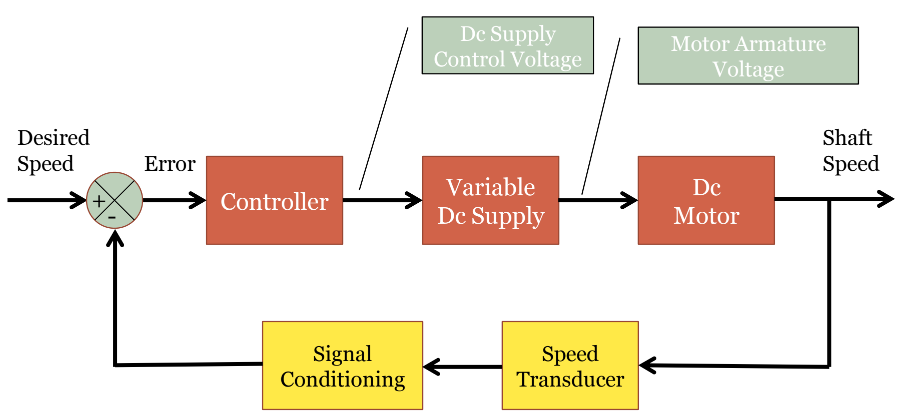
Example: Dc Motor Speed Control



Proportional Control Math Relationship

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Amplify error and send to final control element.

$$C_o = K_p e + C_b$$

Where C_o = the controller output
C_b = the controller output with e =0
K_p = the proportional gain
e = the control error

$$e = SP - Measurement,$$

e can be positive or negative. Error used to take corrective action

e = error signal
SP = setpoint value
Measurement = sensor measurement

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Proportional Control Action

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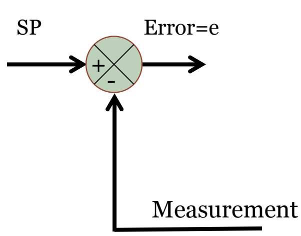

Measurement > SP e negative - output decrease

$$C_o = K_p e + C_b$$

negative

Measurement < SP e positive - output increase

positive

$$C_o = K_p e + C_b$$

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Controller Output Limits

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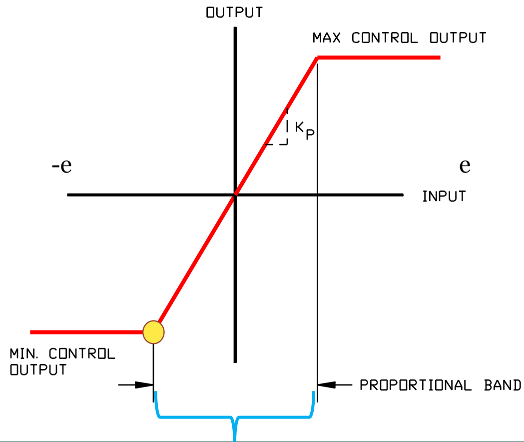
Practical controller output devices have limits on C_o and final control element. Examples: flow value position, motor power supply



Proportional band sets limits of control output.
Determined by value of K_p.

Controller Output Limits

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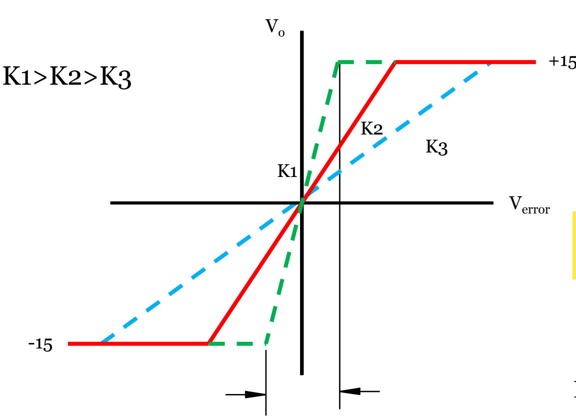
At higher K_p, e produces more correction but reduces the range between controller limits

K1>K2>K3



Band increases as gain decreases

Proportional Band

Computing Proportional Band and K_p

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Proportional gain - change in output /change in error

$$K_p = \frac{\Delta C_o}{\Delta e} = \frac{\Delta \text{Output}}{\Delta(\text{SP} - \text{Measurement})}$$

Increasing proportional gain decreases proportional band (PB). In terms of percent:

$$\%PB = \left(\frac{1}{K_p}\right) \cdot 100\%$$

%PB inversely proportional to K_p

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Proportional Control Action

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Example 9-1: A proportional controller uses an OP AMP with ±15 Vdc output limits when the error input is ±3 Vdc. Saturation of the OP AMP sets these limits. Find the proportional gain and the percent proportional bandwidth from this information. Determine the proportional band and max output if K_p=2.

Find K_p by using $K_p = \frac{\Delta C_o}{\Delta e}$

$$K_p = \frac{C_{max} - C_{min}}{e_{max} - e_{min}}$$

$$K_p = \frac{15-(-15)\ \text{Vdc}}{3-(-3)\ \text{Vdc}}$$

$$K_p = 30/6 = 5 = \frac{\Delta \text{output}}{\Delta e}$$

Find the %PB

$$\%PB = \left[\frac{1}{K_p}\right] 100\%$$

$$\%PB = \frac{1}{5}(100\%) = 20\%$$

20% change in error gives maximum change in output

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Example 9-1 Solution

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Now set the value of proportional gain to 2 and compute the %PB and output

$$\%PB = \frac{1}{K_p}(100\%)$$

$$\%PB = \left(\frac{1}{2}\right)(100\%) = 50\%$$

$$K_p = \frac{\Delta output}{\Delta e} \Rightarrow \Delta output = \Delta e K_p$$

$$\Delta e = 3-(-3) = 6 \qquad \Delta output = \pm 6(2) = \pm 12\,Vdc$$

Lower gain allows the OP AMP to handle more error before saturation.

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Characteristics of Proportional Control

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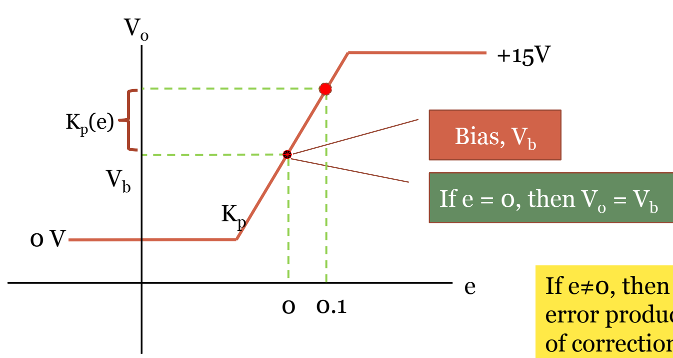
Control formula $V_o = K_p \cdot e + V_b$



If e≠0, then every unit of error produces $K_p(e)$ units of correction that is added/subtracted to V_o.

There is a band of steady-state error about 0 with magnitude of PB% where the output is not saturated.

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Control Offset and Residual Error

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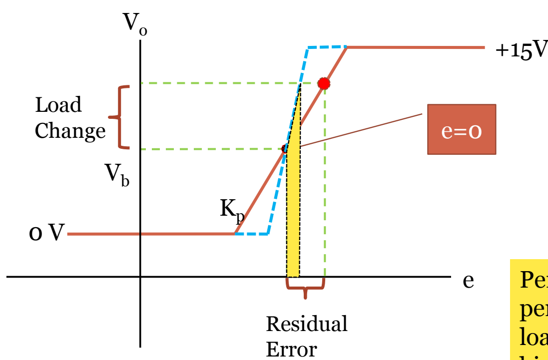
Proportional control always produces a steady-state or residual error when a change in the process load occurs. (When e≠0)



Increasing K_p decreases steady-state error be can not eliminate it.

Permanent load changes produce permanent error. For transient load changes, the error returns to bias value, 0.

Increased Gain & Residual Error

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High gain reduces steady-state error but increases chances of instability

Example 9-2: A proportional controller with a gain of 2 has an output range of 0-15 Vdc for an error input range of 0 to ±1 Vdc. The output has a balance value of 7.5 Vdc. Determine: a.) controller output when error is zero, b.) residual error when a process load change produces and an error voltage of -0.25 Vdc, c.) residual error when control gain increases to 5.

Example 9-2 Solution (1)

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For K_p=2, V_o=C_o=0-15 Vdc, and V_e=e= 0 to ±1 Vdc. Also V_b=C_b=7.5 Vdc.

$C_o = K_p C_e + C_b$ $\quad V_o = K_p V_e + V_b$ $\quad C_o = 2V_e + 7.5$

AT $V_e = 0$ $\quad V_o = 2(0) + 7.5$ V

$V_o = 7.5$ ← Ans

No residual error (Steady-state error)

Process load change produces…

$V_e = -0.25$V

$V_o = 2(-0.25\text{V}) + 7.5$

$V_o = 7.0$

Find residual error….

$K_p = \frac{C_o}{e}$

$e K_p = C_o$

$e = \frac{C_o}{K_p}$

$V_e = \frac{V_o}{K_p}$

$V_e = \frac{7.0}{2} = 3.5$ ← Ans

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Example 9-2 Solution (2)

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For residual (stead-state) error to reach 0, K_p must increase to infinity. Set K_p=5 and compute new residual error

$$\frac{V_o}{K_p} = \frac{7.0}{5} = 1.4\text{Vdc}$$

Residual or steady-state error decreases inversely as the proportional gain increases

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Error As a Function of Proportional Gain

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Computing residual error

$$\%\_error = \frac{SS_d - SS_a}{SS_d}$$

Where: SS_d = desired steady-state output (1 for unit step)
SS_a = actual output at steady-state
%_error= percentage of residual error based on desired value.

Plot the change in residual error as the proportional gain increases using a typical control system

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Error As a Function of Proportional Gain

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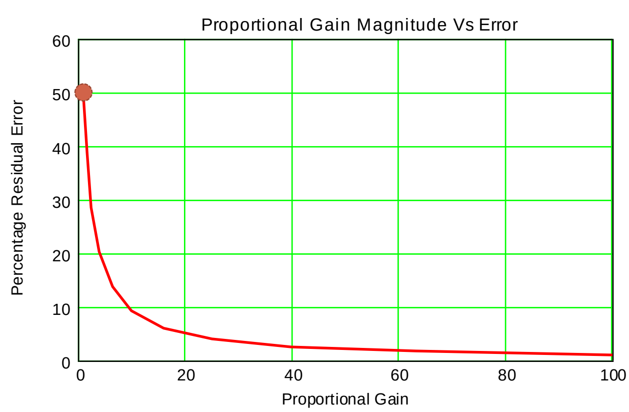

Low values of K_p produce high residual error

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Model of Proportional System (Bias = 0)

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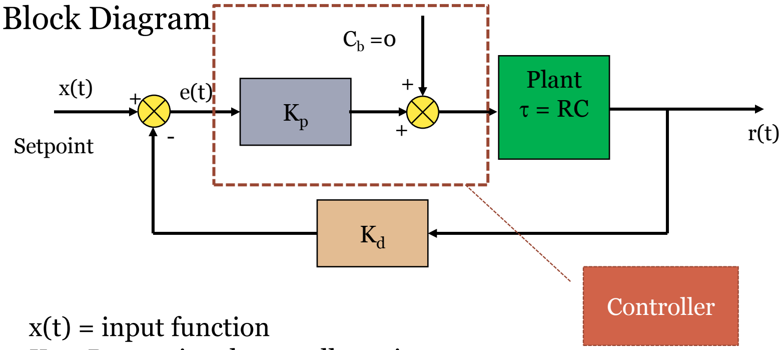
Block Diagram



x(t) = input function
K_p = Proportional controller gain
Plant is modeled using RC circuit
r(t) = output response of the control system
K_d = feedback gain (voltage divider in lab)

Proportional System Time Response

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Method of solution: Signal flow algebra and Laplace transforms
Final solution to step change input:

$$r(t) = \left[\frac{K_p}{1 + K_p \cdot K_d}\right]\left(1 - e^{\frac{-t(1+K_p \cdot K_d)}{R \cdot C}}\right)$$

K_p affects response speed

K_p affects final value

Plot the response of this system to a step input (0-1) for several values of proportional gain and compare them based on response speed and residual error.

Proportional System Time Response

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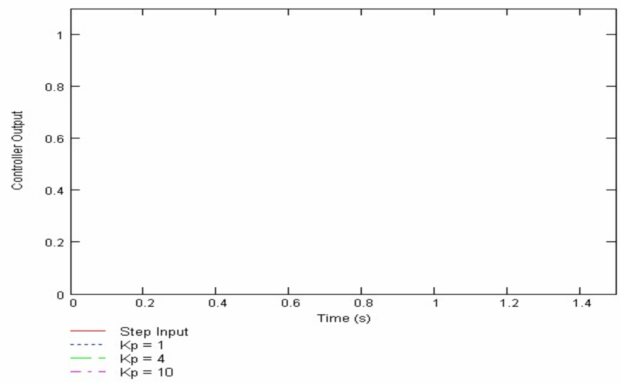

Comparison of response time and residual errors

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End Lesson 9: Proportional Control Action

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